$$\Rightarrow \int_0^1 \frac{t^{2n+2}}{1+t^2}\,dt \leq \int_0^1 t^{2n}\,dt$$

$$\Rightarrow U_n \leq \left[\frac{t^{2n+1}}{2n+1}\right]_0^1$$

$$U_n \leq \frac{1}{2n+1} \quad - 0$$

Donc

$$0 \leq U_n \leq \frac{1}{2n+1}$$

$\forall n \in \mathbb{N}$

On $\lim\limits_{n\to+\infty} \frac{1}{2n+1} = 0$

$$0 \leq U_n \leq \frac{1}{2n+1} \to 0$$

Donc

$$\lim_{n\to+\infty} U_n = 0$$